# FAR BEYOND

# **MAT122**

Integration by Parts (IBP)

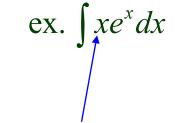


### **Integration by Parts**

Often multiplied functions being integrated don't contain a useful derivative to use *u*-substitution.

Another integration tool is Integration by Parts (IBP).

One function will be labeled u and the other function will be considered a derivative (not of u).



pick the one that simplifies as a derivative to be u

the rest becomes dv

$$\int x \cdot e^x dx$$

$$u \quad dv$$

#### Build a chart:

$$u=x dv = e^x dx$$

$$du = 1 \cdot dx = dx v = e^x$$

#### **Integration by Parts Formula:**

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= xe^{x} - \int e^{x} dx$$
now the integral is easy to take
$$= xe^{x} - e^{x} + C$$

## Integration by Parts – Example #1

ex. 
$$\int xe^{3x}dx$$

$$u dv$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= x \cdot \frac{1}{3}e^{3x} - \int_{3}^{1} e^{3x} dx$$

$$=\frac{1}{3}xe^{3x}-\frac{1}{3}\int e^{3x}dx$$

$$=\frac{1}{3}xe^{3x}-\frac{1}{3}\cdot\frac{1}{3}e^{3x}+C$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

$$u = x dv = e^{3x} dx$$

$$du = dx v = \frac{1}{3}e^{3x}$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int e^{3x} dx$$

$$= \int e^{u} \cdot \frac{1}{3} du$$

$$= \frac{1}{3} \int e^{u} du$$

$$= \frac{1}{3} e^{u}$$

$$= \frac{1}{3} e^{u}$$

$$= \frac{1}{3} e^{u}$$

$$= \frac{1}{3} e^{3x}$$

$$= \frac{1}{3} e^{3x}$$

$$u = 3x$$

$$du = 3dx$$

$$\frac{1}{3}du = dx$$

## **Integration by Parts – Example #2**

ex. 
$$\int x^3 \ln x \, dx$$
 re-order

pick the one that simplifies as a derivative to be *u* 

$$= \int \ln x \cdot x^3 dx = \ln x \cdot \frac{1}{4} x^4 - \frac{1}{4} \int x^4 \cdot \frac{1}{x} dx$$

$$= \lim_{x \to \infty} \frac{1}{4} x^4 - \frac{1}{4} \int x^4 \cdot \frac{1}{x} dx$$
simplify

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4} \int x^3 dx$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4} \cdot \frac{1}{4}x^4 + C$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$

$$u = \ln x \qquad dv = x^3 dx$$

$$du = \frac{1}{x} dx \qquad v = \frac{x^4}{4} = \frac{1}{4} x^4$$

#### **IBP** with One Function

IBP can be used for integrating a single function when it has a derivative but is not a derivative of the function.

ex. 
$$\int \ln x \, dx = \int \ln x \cdot 1 \, dx$$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$
ex. 
$$\int_{2}^{3} \ln x \, dx = x \ln x - x \Big|_{2}^{3}$$

$$= 3 \ln 3 - 3 - (2 \ln 2 - 2)$$

$$= 3 \ln 3 - 3 - 2 \ln 2 + 2$$

$$= 3 \ln 3 - 2 \ln 2 - 1$$
combine like terms
$$= 3 \ln 3 - 2 \ln 2 - 1$$

#### Build a chart:

$$u = \ln x \qquad dv = 1 dx$$

$$du = \frac{1}{2} dx \qquad v = x$$

$$\int u dv = uv - \int v du$$

### **More Integration by Parts**

Sometimes IBP must be taken multiple times:

ex. 
$$\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x dx$$
$$= x^2 e^x - 2 \int x e^x dx$$
$$= x^2 e^x - 2 (x e^x - \int e^x dx)$$
$$= x^2 e^x - 2 (x e^x - e^x) + C$$

$$u = x^{2} dv = e^{x} dx$$
$$du = 2x dx v = e^{x}$$

$$u = x$$
  $dv = e^x dx$   
 $du = dx$   $v = e^x$ 

$$\int u dv = uv - \int v du$$