FAR BEYOND

MAT122

Integration by Parts (IBP)

Stony Brook University

Integration by Parts

Often multiplied functions being integrated don't contain a useful derivative to use u-substitution.

Another integration tool is **Integration by Parts (IBP)**.

One function will be labeled u and the other function will be considered a derivative (not of u).

ex. $\int xe^x dx$

pick the one that simplifies as a derivative to be u

the rest becomes dv

$$\int \underbrace{x}_{u} \cdot \underbrace{e^x dx}_{dv}$$

Build a chart:

$u = x$ $\quad$ $dv = e^x dx$

$du = 1 \cdot dx = dx$ $\quad$ $v = e^x$

Integration by Parts Formula:

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= xe^x - \int e^x dx$$

now the integral is easy to take

$$= \boxed{xe^x - e^x + C}$$

Integration by Parts – Example #1

ex. $\int \underbrace{x}_{u}\underbrace{e^{3x}dx}_{dv}$

$$u = x \quad dv = e^{3x}dx$$
$$du = dx \quad v = \frac{1}{3}e^{3x}$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$= x \cdot \frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}dx$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{3} \cdot \frac{1}{3}e^{3x} + C$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

$$\int u \cdot dv = uv - \int v \cdot du$$

$$\int e^{3x}dx$$

$$u = 3x$$
$$du = 3dx$$
$$\frac{1}{3}du = dx$$

$$= \int e^u \cdot \frac{1}{3}du$$

$$= \frac{1}{3}\int e^u \, du$$

$$= \frac{1}{3}e^u$$

$$= \frac{1}{3}e^{3x}$$

Integration by Parts – Example #2

ex. $\int x^3 \ln x \, dx$

re-order

pick the one that simplifies as a derivative to be u

$$u = \ln x \qquad dv = x^3 dx$$

$$du = \frac{1}{x}\,dx \qquad v = \frac{x^4}{4} = \frac{1}{4}x^4$$

$uv - \int v\,du$

$$= \int \ln x \cdot x^3 dx = \ln x \cdot \frac{1}{4}x^4 - \frac{1}{4}\int x^4 \cdot \frac{1}{x}\,dx$$

simplify

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\cdot\frac{1}{4}x^4 + C$$

$$= \boxed{\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C}$$

IBP with One Function

IBP can be used for integrating a single function when it *has* a derivative but is *not* a derivative of the function.

ex. $\int \ln x \, dx = \int \underbrace{\ln x}_{u} \cdot \underbrace{1 \, dx}_{dv}$

re-write

Build a chart:

$u = \ln x \qquad dv = 1 \, dx$

$du = \frac{1}{x} dx \qquad v = x$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int 1 \, dx$$

$$= \boxed{x \ln x - x + C}$$

$$\int u dv = uv - \int v du$$

ex. $\int_2^3 \ln x \, dx$

$$= x \ln x - x \Big|_2^3$$

$$= 3 \ln 3 - 3 - (2 \ln 2 - 2)$$

$$= 3 \ln 3 - 3 - 2 \ln 2 + 2$$

combine like terms

$$= \boxed{3 \ln 3 - 2 \ln 2 - 1}$$

More Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Sometimes IBP must be taken multiple times:

ex. $\int x^2 e^x \, dx = x^2 e^x - \int e^x \cdot 2x \, dx$

$= x^2 e^x - 2\int x e^x dx$

$uv - \int v du$

$= x^2 e^x - 2(x e^x - \int e^x dx)$

$= \boxed{x^2 e^x - 2(x e^x - e^x) + C}$

$u = x^2$	$dv = e^x dx$
$du = 2x \, dx$	$v = e^x$

$u = x$	$dv = e^x dx$
$du = dx$	$v = e^x$